MICARRAYLIB: SOFTWARE FOR REPRODUCIBLE AGGREGATION, STANDARDIZATION, AND SIGNAL PROCESSING OF MICROPHONE ARRAY DATASETS

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ABSTRACT

micarraylib is a python library to load, standardize, and aggregate datasets collected with different microphone array hardware. The goal is to create larger datasets by aggregating existing and mostly incompatible microphone array data and encoding it into standard B-format ambisonics. These larger datasets can be used to develop novel sound event localization and detection (SELD) algorithms. micarraylib streamlines the download, load, resampling, aggregation, and signal processing of datasets collected with commonly-used and custom microphone array hardware. We provide an API to standardize the 3D coordinates of each microphone array capsule, visualize the placement of microphone arrays in specific spatial configurations, and encode time-series data collected with different microphone arrays into B-format ambisonics. Finally, we also show that the data aggregates can be used to reconstruct a microphone capsule's time-series data using the information from other capsules in the data aggregate. micarraylib will allow for the easy addition of more datasets and microphone array hardware as they become available in the future. All original software written for this paper is released with an open-source license.

Index Terms— sound event detection and localization, microphone arrays, spherical harmonics, ambisonics encoder, multichannel signal processing

1. INTRODUCTION

In the past few years, the field of machine listening has seen major advances in sound event detection (SED) algorithms [1, 2, 3]. These advances have been made possible by the introduction of large datasets with annotated sound events. In particular, the millions of annotated soundclips in AudioSet [4], totaling around 100 hours of data, have been critical for these developments.

In contrast, the development of sound event localization and detection (SELD) algorithms has been slower. This is not surprising, given that SELD datasets are much smaller than AudioSet, usually with only a few thousand sound events with annotations for both category and spatial localization (example datasets include those introduced by the DCASE SELD challenges in 2019 [5], 2020 [6], and 2021 [7], as well as the LOCATA challenge [8]).

To develop SELD algorithms that are as robust as existing SED ones, machine listening researchers will need access to large amounts of data collected with microphone arrays. A number of publicly available microphone array datasets exist (see Table 1), but these datasets have heterogeneous hardware parameters, thus complicating their aggregation. Table 1: Some publicly available microphone array datasets. The number of microphone arrays, total microphone capsules, length in hours, and presence of SELD annotations are tabulated.

dataset	no. arr	capsules	length	SELD
DCASE(3) 2019 [5]	1	4	8 Hr	Yes
DCASE(3) 2020 [6]	1	4	13 Hr	Yes
DCASE(3) 2021 [7]	1	4	13 Hr	Yes
LOCATA [8]	4	63	0.5 Hr	Yes
3D-MARCo [9]	7	71	0.2 Hr	No
EigenScape [10]	1	32	11 Hr	No

Two notorious differences between microphone array datasets include 1) the use of different microphone hardware and 2) conventions for SELD annotation (or complete lack of), including event start time, duration, and position in space. To standardize microphone array recordings across different hardware, some researchers encode them into the ambisonics B-format [11, 12], which uses the individual microphone capsule coordinates to compute a matrix of spherical harmonic coefficients. In its simplest form, the B-format is obtained by multiplying the pseudo-inverse of this matrix by the corresponding raw capsule recordings (also known as A-format) [13]. The ambisonics B-format captures specific spatial features (i.e. the first channel is equivalent to an omnidiectional microphone, the next three channels are fig-8 microphones aligned on the x, y, and z Cartesian coordinates, etc. see [14] for details on the spherical harmonics theory that results in B-format encoding). On the other hand, standardizing SELD annotations is possible if certain parameters (i.e. event start time, end time, and a position in space) are parsed to be consistent across datasets. Additionally, since not all microphone array datasets have SELD annotations, the use of unsupervised and self-supervised learning strategies to learn spatial representations will be necessary to use all available datasets.

Here we introduce micarraylib, a python library to download, standardize, and aggregate existing microphone array recordings. Using micarraylib, one can encode raw microphone array recordings across different datasets to be in the common ambisonics B-format. micarraylib also standardizes annotations to be in a common convention. Additionally, micarraylib organizes metadata (i.e. microphone capsule coordinates and hardware name) to be readily accessible. micarraylib is freely available at https://github.com/micarraylib/micarraylib.

In the next sections we describe micarraylib's functional principles and show example applications, which include the aggregation of different SELD datasets, visualization of aggregated microphone coordinates, and data augmentation via interpolation of a

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virtual capsule recording using data from neighboring capsules.

2. LIBRARY FUNCTION

We standardize three elements present in most microphone array datasets: 1) metadata, 2) SELD labels (if any), and 3) audio format.

2.1. Metadata processing

The most important piece of metadata accompanying any microphone array dataset is its microphone capsule coordinates. Microphone array datasets often use different microphone hardware. Designers publish the relative distance between microphone capsules and a reference point. These can be converted to 3D coordinates (Cartesian or polar). In some datasets the reference point is the microphone array's center, or a specific location in a physical space. micarraylib standardizes these distances and locations to be a common coordinate format (3D vectors, either Cartesian or polar). These coordinates can be used to visualize microphone arrays, to compute spherical harmonics, and more generally to develop algorithms that incorporate spatial information at the level of microphone capsule location.

Other pieces of metadata that micarraylib processes (if available) and makes available to the user include sound scene category, musical artist, and geographic location of the recording.

2.2. SELD label standardization

Because of their unique ability to capture spatial information, microphone arrays are a common hardware choice to collect SELD data. While several labeling conventions exist among datasets, annotations for sound events include at least a start time, but may also have an end time, and a position in space. micarraylib standardizes sound event labels across datasets to have the following format: start and end time (python tuple), object category (a unique integer or string), location coordinates (3D vector that may change over a fixed time-step if the event moves), and active time-steps (list of booleans if the event is transiently on or off). When one of these parameters does not exist for an event, micarraylib will indicate it with a None. By using micarraylib, researchers will access microphone array datasets with a standard format for spatial sound event labels.

2.3. Audio standardization

The ambisonics B-format allows for the standardization of microphone array recordings [11, 12]. A simple ambisonics encoder uses the individual microphone capsule coordinates to compute a matrix of spherical harmonics. The pseudo-inverse of this matrix then multiplies the raw capsule recordings to encode them into B-format channels.

While more complicated encoders are usually proprietary and include multiple effects that aid perceptual parameters [15], computing the spherical harmonics using microphone array coordinates is a straightforward operation if the microphone capsule coordinates are known. The *N*th-order spherical harmonic matrix can be computed using equation 1.

$$Y_{n,l}(\theta,\phi) = X_{n,|l|} P_{n,|l|} \cos(\theta) \begin{cases} \sqrt{2} \sin(|l|\phi) & \text{if } l < 0\\ 1 & \text{if } l = 0 \\ \sqrt{2} \cos(l\phi) & \text{if } l > 0 \end{cases}$$
(1)

This is the conventional equation used in the field of acoustics to compute the Laplace spherical harmonics [16]. n indexes the spherical harmonic order (i.e. 0th, 1st, 2nd, ..., Nth) and mindexes the degree (i.e. each order n has degrees $m \in [-n, n]$ degrees). $\theta \in [0, \pi]$ is the vertical angle advancing from top to bottom, $\phi \in [0, 2\pi]$ is the azimuth angle starting at the front of the microphone array and advancing counter-clockwise. $X_{n,|m|}$ is a normalization factor that ensures that spherical harmonics have unit magnitude [16] and $P_{n,|m|}$ is the Legendre function (without the Condon-Shortley phase) [17].

micarraylib converts raw capsule recordings into the common ambisonics B-format. This results in audio signals that have shared spatial characteristics across channels, independent of which microphone hardware was used to collect them.

3. LIBRARY ORGANIZATION AND COPYRIGHT

micarraylib is written in python and all its contents are opensource. The following subsections describe the organization of its file structure as seen from its root directory.

3.1. Micarrays

Directory that contains the array_shapes_raw.py and array_directions_raw.py files, which list the raw (as released by the manufacturer) shape (i.e. coordinates) and capsule directionality of each microphone array supported. The names that the manufacturer gave to each capsule in the microphone array are also included in these files. The micarray.py file in this directory defines a micarray object with attributes that summarize all the information provided by the manufacturer.

This directory also has files with functions that process the data from the array_shapes_raw.py and array_directions_raw.py files and standardize it to be in 3D Cartesian and/or polar coordinates.

3.2. Util

Directory that contains a utils.py file with basic functionalities for dataset standardization, such as functions to convert between polar and Cartesian coordinates, normalize units of length to meters and radians, and normalize time units to seconds (in SELD labels, for example). It also contains a plotting.py file with functions that tailor matplotlib's plotting for microphone arrays.

3.3. Encoder

File defining the encoder object with its main attribute being a set of capsule coordinates (used to calculate the matrix of spherical harmonics). It also has an encode method that takes a numpy array with raw recordings and returns a simple encoding of these recordings in ambisonics B-format.

3.4. Dataset

File that defines the dataset object using the soundata API [18]. soundata is a new python library with tools to download and load common audio datasets with corresponding annotations and metadata. In addition to soundata attributes, the dataset object includes a list of microphone capsule coordinates used. The soundata API includes all methods to download and load the

original data. micarraylib standardizes the SELD annotations when they are not standardized by soundata.

3.5. Aggregator

File that defines the aggregate_datasets object, whose attributes include a list of dataset objects. Its default method standardizes all recordings across datasets to be the same number of channels in ambisonics B-format, and pairs individual recordings with their corresponding SELD labels.

It also defines the micarray_aggregate object, which aggregates coordinates and recordings across microphone arrays (useful when multiple pieces of microphone array hardware are used together in a single dataset, such as the 3D-MARCo or LOCATA datasets).

3.6. Augmentation

File that defines a data_augmentation object, which uses a neural network model to virtually add capsule recording data to a microphone array dataset at a coordinate defined by the user. Section 5 below describes the current functionality of this model (which is limited to reconstruction of channels within the EigenMike [19] hardware at the time of this writing, but we are working to expand its possibilities).

3.7. Copyright

micarraylib is released with a Creative Commons License. We also do not alter any dataset's license, as micarraylib only accesses data already hosted online (via soundata; as a result, datasets are not redistributed by micarraylib).

4. AGGREGATING DATASETS

micarraylib streamlines the aggregation of existing microphone array datasets. Figure 1 shows the code needed to standardize and aggregate the six different datasets in Table 1.

One at a time, micarraylib separately encodes each recording into a first-order ambisonics B-format (4 channels total; the first-order ambisonics limit is determined by the dataset with the lowest number of raw capsule recordings: 4 channels in the DCASE SELD datasets). After the simple encoding step, we have a total of 46 hours of audio data in a common ambisonics Bformat. micarraylib also standardizes the SELD labels from the DCASE SELD and LOCATA datasets to have a start and end time, object category, spatial coordinates, and active time-steps. The 3D-MARCo and EigenScape datasets do not have SELD labels, and the resulting aggregate indicates this with None entries in the label attribute for those specific recordings. In the end, 34 hour of data in this dataset aggregate have labeled sound events.

4.1. Hardware considerations and next steps

Aggregating different datasets can result in SELD methods that confound elements that are different between datasets (i.e. hardware, events, and/or ambient). While our library encodes all datasets into the standard ambisonics B-format, it is important to keep in mind that the hardware differences between datasets could remain in the B-format. For this reason, we plan to continue fine-tuning our encoder to quantify and reduce differences between hardware. To better quantify these effects, datasets with scenes and events that

```
1 import micarraylib as mc
  datadir = ' ~/datasets'
5
  datasets = [
      mc.datasets.dcase19(datadir),
6
      mc.datasets.dcase20(datadir).
      mc.datasets.dcase21(datadir),
      mc.datasets.locata(datadir),
9
10
      mc.datasets.marco(datadir).
      mc.datasets.eigenscape(datadir)
11
12 ]
  for dataset in datasets:
14
      dataset.load() # using the soundata API [18]
16
  aggregate = mc.aggregators.aggregate_datasets(
17
18
      datasets,
      sr=24000.
19
20
 )
```

Figure 1: Downloading, loading, and aggregating the six datasets in Table 1 using micarraylib.

are simultaneously collected with different hardware (i.e. the 3D-MARCo and LOCATA datasets) will be particularly useful.

5. DATA AUGMENTATION

An idealized spatial recording of a sound scene would record information at all locations in the space continuum. Since such idealized scenario is not possible with existing hardware, researchers must sample specific locations using microphone arrays with capsules at specific coordinates. micarraylib includes a model can be used to virtually add microphones to a dataset via interpolation from existing microphone capsule data.

5.1. Technical motivation

Aggregating microphone arrays can lead to denser spatial sampling of a sound scene. The resulting dense samplings of a sound scene are redundant [20]. Therefore, given a set of microphone capsules recording a common scene or source, it should be possible to interpolate, with some error, one of the capsule's time-series using the recordings collected with all other capsules. If this is possible, it should also be possible to virtually generate the recording of a microphone capsule outside but near the microphone array topology.

While a detailed empirical study of virtual microphone capsule time-series generation deserves a separate scientific report, micarraylib already includes some of this functionality. Here we describe a series of experiments that we carried out to design a model able to virtually add the recording of a missing microphone capsule using the recordings from other capsules in the Eigen-Mike microphone array. These experiments also show the utility of micarraylib to aggregate datasets that can then be used for machine listening research.

In all experiments we ask the question: can the recording of a microphone capsule be reconstructed given the preceding 5 milliseconds of recordings with neighboring capsules? We hypothesize that such reconstruction is possible using both the recordings from neighboring capsules and their 3D spatial coordinates.

Table 2: Model performance for capsule interpolation experiments

Model	MSE (eval)
before training	0.9
exp 1	0.000039
exp 2	0.00013
exp 3	0.0016

5.2. Data

We used micarraylib to aggregate the EigenMike data from the EigenScape, 3D-MARCo, and LOCATA datasets. We skip the encoding step and keep the recordings in the raw format (A-format). We split each recording into development (the first 80% frames) and testing (the last 10% frames) subsets (the frames between the %80 and 90% of each recording length were left out to minimize the effects of temporal correlation between the development and testing data).

5.3. Methodology

The input features to train our model are an intermediate step between A-format and B-format that is computed as follows. We calculate the fourth order spherical harmonic matrix using the 3D coordinates for each EigenMike capsule and equation (1). The result is a matrix W that has $(N+1)^2$ rows (N = 4) and 32 columns, one corresponding to each EigenMike capsule. We multiply the sample recorded by a given capsule with the spherical harmonic coefficients in the corresponding column of matrix W. We stack 20 of these matrices into a tensor that corresponds to 5 consecutive milliseconds of samples in a recording. We refer to these concatenated matrices as tensor X.

In the experiments described below we remove some audio information from tensor X and we train a simple LeNet CNN [21] to use X to reconstruct the recording of a single capsule (whose audio information is missing from X) with a MSE loss function.

5.4. Experiments and results

We carried out three experiments: 1) Tensor X is only missing the audio information of the single capsule that we want to reconstruct. 2) The samples of five neighboring capsules are also removed from X. 3) X only has the audio information of the three capsules that would result in a tetrahedral geometry (4 capsules total) with respect to the capsule whose recording we want to reconstruct.

We trained a LeNet CNN model from scratch using the ADAM optimizer with a patience criterion of 1000 epochs. For each of these three experiments, the model's task was to reconstruct the missing samples of a single microphone capsule in the EigenMike using X as input. The average MSE across datapoints in the evaluation set was computed after training and is shown in Table 2.

As shown in Table 2, the evaluation MSE before training was close to 1. Experiment 1 reduced this MSE by 5 orders of magnitude, showing that a single capsule's samples can be reconstructed (with some error) using all other 31 capsule recordings in the Eigen-Mike. Experiment 2 also reduced the MSE but only by 4 orders of magnitude, showing that reconstruction of the same capsule's recording is affected by the fact that audio information from neighboring capsules was missing. Experiment 3 only reduced the MSE by 3 orders of magnitude due to there only being information from



Figure 2: Interactive visualization with micarraylib of the location and directionality of microphone arrays used to collect the LOCATA dataset, one of which is the EigenMike. Using matplotlib's 3D plotting, micarraylib includes methods to plot capsules in microphone array hardware.

3 other microphone capsules to reconstruct the capsule's recording. These results show that reconstructing a capsule's samples using other capsule's recordings is possible, but the interpolation is sensitive to the density and proximity of recordings available to carry out this reconstruction.

5.5. Next steps

Given that reconstructing a microphone's signal seems possible within EigenMike capsules, we will train a model able to use Eigen-Mike data to recover the signal captured by a capsule or array outside the EigenMike geometry. For this purpose, we will use the microphone array aggregates from the 3D-MARCo and LOCATA datasets, which include data recorded by microphone capsules outside the EigenMike structure but very near to it (see Fig 2). Our goal is for micarraylib to ultimately allow for the virtual simulation of microphone capsules in arbitrary coordinates.

The idea of virtual microphone capsule generation has already been explored with supervised learning using neural network techniques like CNNs [22, 23] or autoencoders [24], via statistical interpolation with β -divergence [25] or with pure signal processing [26]. The full potential of the microphone array data augmentation that we propose will be achieved by using micarraylib in combination with audio augmentation libraries like MUDA [27] and Audiomentations (https://github.com/iver56/audiomentations).

6. CONCLUSIONS AND FUTURE DIRECTIONS

We have presented micarraylib, a library to aggregate microphone array datasets by standardizing microphone array coordinates, SELD labels, and recordings using a simple ambisonics Bformat encoder. Our presentation of the library also included practical demonstrations that show both the utility of micarraylib and the need for it in the field of machine listening. Researchers will be able to use micarraylib to create large aggregates of standardized microphone array datasets. We are also looking forward to seeing what other functionalities the machine listening research community believes should be added to micarraylib. We will continue adding new datasets to micarraylib in years to come, and we will also add support for more complex signal processing procedures that include completely custom array shapes, microphone polarity, methods for perceptually-plausible ambisonics encoding and decoding, as well as processing of motion-capture data for moving sound events and microphone arrays.

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